CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP232 MATHEMATICS FOR COMPUTER SCIENCE

ASSIGNMENT 2 FALL 2015

PROBLEM 2a: Direct proof: assume that the LHS is True. Thus each of the following is True:

(1) $(\neg p \vee q) \to r$ (2) $s \vee \neg q$ (3) $\neg t$ (4) $p \to t$ (5) $(\neg p \wedge r) \to \neg s$

From (3) it follows that t is False.

From (4) it then follows that p is False.

From (1) it then follows that r is True.

From (5) it then follows that s is False.

From (2) it then follows that q is False.

PROBLEM 2b: Direct proof: assume that the LHS is True. Thus each of the following is True:

(1) $\neg p \to (r \wedge \neg s)$ (2) $t \to s$ (3) $u \to \neg p$ (4) $\neg w$ (5) $u \vee w$

From (4) it follows that w is False.

From (5) it then follows that u is True.

From (3) it then follows that p is False.

From (1) it then follows that r is True and s is False.

From (2) it then follows that t is False.

Hence $\neg t \vee w$ is True.

PROBLEM 2c: Direct proof: assume that the LHS is True. Thus each of the following is True:

(1) $p \vee q$ (2) $q \to r$ (3) $(p \wedge s) \to t$ (4) $\neg r$ (5) $\neg q \to (u \wedge s)$

From (4) it follows that r is False.

From (2) it then follows that q is False.

From (1) it then follows that p is True.

From (5) it then follows that u and s are True.

From (3) it then follows that t is True.

PROBLEM 3a: Direct proof:

Suppose that (1) $\forall x[R(x) \rightarrow (S(x) \vee Q(x))]$ and (2) $\exists x[\neg S(x)]$ are True.

From (2), it follows that $S(x_1)$ is False for some x_1.

Next, note that (1) is equivalent to $\forall x[\neg R(x) \vee S(x) \vee Q(x)]$.

Thus, in particular $\neg R(x_1) \vee S(x_1) \vee Q(x_1)$ is True.

Since $S(x_1)$ is False it follows that $\neg R(x_1) \vee Q(x_1)$ must be True, *i.e.*, $R(x_1) \rightarrow Q(x_1)$ is True.

Thus $\exists x[R(x) \rightarrow Q(x)]$ is True, namely for $x = x_1$.

PROBLEM 3b: Proof by contradiction:

Suppose that (1) $\forall x[P(x) \vee Q(x)]$ and (2) $\forall x[(\neg P(x) \wedge Q(x)) \rightarrow R(x)]$ are True,

but that the conclusion (3) $\forall x[\neg R(x) \rightarrow P(x)]$ is False.

If (3) is False it then $\neg R(x_1) \rightarrow P(x_1)$ is False for some x_1.

Equivalently, we have that $R(x_1) \vee P(x_1)$ is False for some x_1.

Thus both $P(x_1)$ and $R(x_1)$ are False.

From (2) it then follows that $Q(x_1)$ must be False.

Hence both $P(x_1)$ and $Q(x_1)$ are False, which contradicts (1).